Renormalization group analysis of the anisotropic nonlocal Kardar-Parisi-Zhang equation with spatially correlated noise

Youngkyun Jung, Kwangho Park, Hyun-Joo Kim, and In-mook Kim
Department of Physics, Korea University, Seoul 136-701, Korea

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We study an anisotropic nonlocal Kardar-Parisi-Zhang (KPZ) equation with spatially correlated noise by using the dynamic renormalization group method. When the signs of nonlinear terms in parallel and perpendicular directions are opposite, the correlated noise coupled with the long ranged nature of interaction produces a stable non-KPZ fixed point for $d<d_c$. For the uncorrelated noise, the roughness and dynamic exponents associated with the stable fixed point are different from those of the isotropic nonlocal KPZ equation, while for the correlated noise the exponents are the same as those of the isotropic case.

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For a long time the kinetic roughening of surface has attracted much interest because it is related to various physical phenomena such as crystal growth, bacterial growth, paper wetting, molecular beam epitaxy, etc. [1]. An interesting feature of the kinetic roughening of a surface is that the global surface width shows a nontrivial scaling behavior. The global surface width is defined by $W(L,t)=\langle L^{-d}\Sigma_i[h_i(t)-\overline{h}(t)]^2\rangle^{1/2}$. The surface width scales as $W\sim t^{\alpha/z}$ for $t\ll L^z$, and $W\sim L^\alpha$ for $t\gg L^z$. Here $\overline{h},L,d$, and $h_i(t)$ denote the mean height, the system size, the substrate dimension, and the height at time t and site i, respectively. α, z, and $\beta(=\alpha/z)$ are the roughness, the dynamic, and the growth exponent, respectively. A well-known continuum equation for the kinetic roughening of a surface is the Kardar-Parisi-Zhang (KPZ) equation [2],

$$\frac{\partial h(x,t)}{\partial t}=\nu\nabla^2 h+\frac{\lambda}{2}(\nabla h)^2+\eta(x,t). \tag{1}$$

The noise $\eta(\mathbf{r},t)$ satisfies $\langle\eta(\mathbf{k},\omega)\rangle=0$, and $\langle\eta(\mathbf{k},\omega)\eta(\mathbf{k}',\omega')\rangle=2D\delta^d(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega')$, where $\eta(\mathbf{k},\omega)$ is the Fourier transform of noise $\eta(\mathbf{r},t)$ and d is the substrate dimension. The KPZ equation has a nonlinear term of short-range describing the lateral growth. The KPZ equation without a nonlinear term is called the Edwards-Wilkinson equation [3]. The conserved version of the KPZ equation is a conserved KPZ equation [4,5].

The applicability of the KPZ equation seems to encompass a variety of growth phenomena, but KPZ behavior is not observed in many experiments about the surface growth. Recently it was proposed that incorporating the long-ranged nature of interactions is necessary for a wide class of problems such as the long-ranged hydrodynamic interactions, proteins, colloids, etc. [6–10]. Phenomenological equations in the presence of long-range interactions, the nonlocal KPZ (NKPZ) equation [11] and the nonlocal conserved KPZ (NCKPZ) equation [12] were introduced. The long-range effect of the nonlinear terms in both equations is introduced by coupling the gradients at two different points. It was found that, due to the long-ranged nature, the roughness of the surface is changed, and several distinct phases appear. The effect of spatially correlated noise on the NKPZ and NCKPZ equations was also studied [13,14]. The correlated noise, coupled with the long-range interactions, produces new fixed points at which the roughness of the surface depends on both the long-range interaction strength and the spatially correlation parameter.

On the other hand, it is well known that the critical exponents can also be modified by an anisotropic nature of the substrate [15]. Wolf [15] studied the anisotropic KPZ (AKPZ) equation, which can be applied to various physical growth problems such as ion-sputtered surface growth [16], and surface growth on the reconstructed surface structure [17]. The AKPZ equation is written as

$$\frac{\partial h(\mathbf{r},t)}{\partial t}=\nu_\parallel\nabla_\parallel^2 h+\nu_\perp\nabla_\perp^2 h+\frac{\lambda_\parallel}{2}(\nabla_\parallel h)^2+\frac{\lambda_\perp}{2}(\nabla_\perp h)^2+\eta(\mathbf{r},t), \tag{2}$$

where $\nabla_\perp(\nabla_\parallel)$ is the gradient along the perpendicular (parallel) direction. The anisotropy means $r_\nu\equiv\nu_\parallel/\nu_\perp\neq 1$ and $r_\lambda\equiv\lambda_\parallel/\lambda_\perp\neq 1$. From studying the AKPZ equation with white noise in $2+1$ dimensions by using the dynamic renormalization group (RG) method, Wolf found that when the signs of λ's are opposite ($r_\lambda<0$), the nonlinear terms turn out to be irrelevant under the RG transformation. As a result, the AKPZ equation with opposite signs of λ's belongs to the Edwards-Wilkinson (EW) universality class.

To our knowledge, there is no study about how the presence of anisotropy changes the physical properties of the NKPZ equation with spatially correlated noise. In this paper, we study the anisotropic NKPZ (ANKPZ) equation with spatially correlated noise by using the dynamic RG method. The ANKPZ equation with spatially correlated noise is written as

$$\frac{\partial h(\mathbf{r},t)}{\partial t}=\nu_\parallel\nabla_\parallel^2 h(\mathbf{r},t)+\nu_\perp\nabla_\perp^2 h(\mathbf{r},t)+\eta(\mathbf{r},t)+\sum_{\Psi=\parallel,\perp}\int d\mathbf{r}'\frac{1}{2}\vartheta_\Psi(\mathbf{r}')\times\nabla_\Psi h(\mathbf{r}+\mathbf{r}',t)\cdot\nabla_\Psi h(\mathbf{r}-\mathbf{r}',t). \tag{3}$$

The noise $\eta(\mathbf{r},t)$ satisfies $\langle\eta(\mathbf{k},\omega)\rangle=0$, and

TABLE I. Table of the effective coupling constants. For example, $U_{\rho 0}$ is the coupling constant, where long-ranged interaction couples with white noise.

Coupling constant	Interaction	Noise
U_{00}	Short range	White noise
$U_{\rho 0}$	Long range	White noise
$U_{0\sigma}$	Short range	Correlated noise
$U_{\rho\sigma}$	Long range	Correlated noise

$$\langle \eta(\mathbf{k},\omega)\eta(\mathbf{k}',\omega')\rangle = 2D(\mathbf{k},\omega)\delta^d(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega'). \tag{4}$$

For spatially correlated noise, the form of $D(\mathbf{k},\omega)$ can be written as $D_0+D_\sigma k^{-2\sigma}$. Here σ is an exponent characterizing the decay of spatial correlations. The kernel $\vartheta_\perp(\mathbf{r})$ and $\vartheta_\parallel(\mathbf{r})$ have a short-ranged part $\lambda_{0\Psi}\delta^d(\mathbf{r})$ and a long-ranged part $\lambda_{\rho\Psi}r^{\rho-d}$. In Fourier space, $\vartheta_\Psi(\mathbf{k})=\lambda_{0\Psi}+\lambda_{\rho\Psi}k^{-\rho}$, where Ψ are $\perp$ and $\parallel$. We consider only the case of positive $\nu_\perp$ and $\nu_\parallel$ for stable surfaces. In this study, when the signs of λ's are opposite, we found that a non-NKPZ fixed point exists in the parameter space of the effective coupling constants if $d<d_c$. However, when the signs of λ's are the same, the anisotropy becomes irrelevant, and thus the scaling behavior is the same as that of the isotropic NKPZ equation [13].

The calculations of the RG transformation were performed by combining two methods, one of which was used by Chattopadhyay to study the NKPZ equation with correlated noise [13], and the other by Jeong *et al.* to study the AKPZ equation with correlated noise [18]. The steps of the RG transformation are as follows. First, the anisotropic exponent χ is introduced to relate two characteristic length scales as $\xi_\parallel\sim\xi_\perp^\chi$, where $\xi_\parallel$ and $\xi_\perp$ are characteristic length scales in parallel and perpendicular directions on the substrate, respectively. Next, a coarse-graining transformation is performed within the one-loop order by integrating out the fluctuations of heights within small length scales [19], which correspond to the wave vectors $e^{-\ell}\pi/a\leqslant|\mathbf{k}_\perp|\leqslant\pi/a$ and $e^{-\ell\chi}\pi/a\leqslant|\mathbf{k}_\parallel|\leqslant\pi/a$ (a is the lattice constant), for the parameters $\nu_\perp$, $\nu_\parallel$, $\lambda_\perp$, $\lambda_\parallel$, and D and the effective coupling constants U_{00}, $U_{\rho 0}$, $U_{0\sigma}$, and $U_{\rho\sigma}$. The effective coupling constants are defined as $U_{xy}^2=K_{d-1}(\lambda_{x\perp}^2D_y)/2\pi\nu_\perp^3$, where x are 0 and ρ, and y are 0 and σ (see Table I). Here $K_d=S_d/(2\pi)^d$, and S_d is the surface area of a d-dimensional unit sphere. Finally, the rescalings are performed as $x_\perp\to e^\ell x_\perp$, $x_\parallel\to e^{\ell\chi}x_\parallel$, $h\to e^{\ell\alpha}h$, and $t\to e^{\ell z}t$. The rescaling leads to $r_{0\lambda}(\equiv\lambda_{0\parallel}/\lambda_{0\perp})\to e^{2\ell(1-\chi)}r_{0\lambda}$ and $r_{\rho\lambda}(\equiv\lambda_{\rho\parallel}/\lambda_{\rho\perp})\to e^{2\ell(1-\chi)}r_{\rho\lambda}$. From the scale invariance, $\chi=1$ is obtained, provided that $r_{0\lambda}\neq 0$ and $r_{\rho\lambda}\neq 0$. The scaling relations $\alpha+z=2$ and $\alpha+z=2-\rho$ are obtained from the scale invariance of $\lambda_{0\Psi}$ and $\lambda_{\rho\Psi}$, respectively. For $\chi=1$, the recursion relations are explicitly calculable, but could not be done for $\chi\neq 1$. Hence we consider the recursion relations for $\chi=1$ and $r_\nu>0$.

We find the RG recursion relations for $\chi=1$ as

$$\frac{d\nu_\perp}{d\ell}=\nu_\perp\{z-2-(\bullet)\}, \tag{5}$$

$$\frac{dr_\nu}{d\ell}=r_\nu\left\{(\bullet)-\frac{1}{r_\nu}(\bigcirc)\right\}, \tag{6}$$

$$\frac{dU_{00}}{d\ell}=U_{00}\frac{2-d}{2}+\frac{(U_{00}^2+U_{0\sigma}^2)}{2U_{00}}(\star)+\frac{3}{2}U_{00}(\bullet), \tag{7}$$

$$\frac{dU_{\rho 0}}{d\ell}=U_{\rho 0}\frac{2-d+2\rho}{2}+\frac{(U_{\rho 0}^2+U_{\rho\sigma}^2)}{2U_{\rho 0}}(\star)+\frac{3}{2}U_{\rho 0}(\bullet), \tag{8}$$

$$\frac{dU_{0\sigma}}{d\ell}=U_{0\sigma}\frac{2-d+2\sigma}{2}+\frac{3}{2}U_{0\sigma}(\bullet), \tag{9}$$

$$\frac{dU_{\rho\sigma}}{d\ell}=U_{\rho\sigma}\frac{2-d+2\rho+2\sigma}{2}+\frac{3}{2}U_{\rho\sigma}(\bullet), \tag{10}$$

where each symbol is given by

$$\begin{aligned}(\bullet)=&(U_{00}^2+U_{0\sigma}^2)\left(A_0-\frac{2C_0}{d-1}\right)+(U_{00}U_{\rho 0}+U_{0\sigma}U_{\rho\sigma})\\&\times\left[\left(A_0-\frac{2C_0}{d-1}\right)+2^{-\rho}\left(A_\rho-\frac{2C_\rho}{d-1}\right)-\frac{3\rho B_0}{d-1}\right]\\&+2^{-\rho}(U_{\rho 0}^2+U_{\rho\sigma}^2)\left(A_\rho-\frac{2C_\rho}{d-1}-\frac{3\rho B_\rho}{d-1}\right)-\frac{2\sigma}{d-1}\\&\times[U_{0\sigma}^2B_0+U_{0\sigma}U_{\rho\sigma}(B_0+2^{-\rho}B_\rho)+2^{-\rho}U_{\rho\sigma}^2B_\rho],\end{aligned} \tag{11}$$

$$\begin{aligned}(\star)=&(U_{00}^2+U_{0\sigma}^2)(C_0+r_{0\lambda}E_0)+2^{1-\rho}(U_{00}U_{\rho 0}+U_{0\sigma}U_{\rho\sigma})\\&\times(C_0+r_{\rho\lambda}E_0)+2^{-2\rho}(U_{\rho 0}^2+U_{\rho\sigma}^2)(C_\rho+r_{\rho\lambda}E_\rho),\end{aligned} \tag{12}$$

$$\begin{aligned}(\bigcirc)=&(U_{00}^2+U_{0\sigma}^2)r_{0\lambda}(A_0-2r_\nu E_0)+(U_{00}U_{\rho 0}+U_{0\sigma}U_{\rho\sigma})\\&\times[r_{\rho\lambda}(A_0-2r_\nu E_0)+2^{-\rho}r_{0\lambda}(A_\rho-2r_\nu E_\rho)\\&-3\rho r_{\rho\lambda}F_0]+2^{-\rho}(U_{\rho 0}^2+U_{\rho\sigma}^2)[r_{\rho\lambda}(A_\rho-2r_\nu E_\rho)\\&-3\rho r_{\rho\lambda}F_\rho]-2\sigma[U_{0\sigma}^2r_{0\lambda}F_0+U_{0\sigma}U_{\rho\sigma}(r_{\rho\lambda}F_0\\&+2^{-\rho}r_{0\lambda}F_\rho)+2^{-\rho}U_{\rho\sigma}^2r_{\rho\lambda}F_\rho].\end{aligned} \tag{13}$$

A_x, B_x, C_x, E_x, and F_x in Eqs. (12)–(14) are given by

$$\begin{aligned}&A_x=H+r_{x\lambda}I,\quad B_x=E+r_{x\lambda}F,\\&C_x=A+r_{x\lambda}B,\quad E_x=B+r_{x\lambda}C,\\&F_x=F+r_{x\lambda}G,\end{aligned} \tag{14}$$

where x is 0 or ρ. A, B, C, E, F, G, H, and I in Eq. (14) are $A=Q(0,(d-2)/2,3)$, $B=Q(2,(d-2)/2,3)$, $C=Q(4,(d-2)/2,3)$, $E=Q(0,d/2,2)$, $F=Q(2,d/2,2)$, $G=Q(4,d/2,2)$, $H=Q(0,(d-2)/2,2)$, and $I=Q(2,(d-2)/2,2)$, respectively, where

$$Q(\alpha,\beta,\gamma)\equiv\int_0^\infty dy\,\frac{y^\alpha}{(1+y^2)^\beta(1+r_\nu y^2)^\gamma}. \tag{15}$$

Now let us consider the following four sets of coupling constants: $(U_{00},U_{\rho 0})$,$(U_{0\sigma},U_{\rho\sigma})$, $(U_{00},U_{0\sigma})$, and $(U_{\rho 0},U_{\rho\sigma})$. From the RG recursion relations, we found on- or off-axial stable fixed points for the four sets of parameters we considered. In each set, there exist stable fixed points for $r_{x\lambda}>0$ and $r_{x\lambda}<0$ (x is 0 or ρ) if $d<d_c$. In the case of $r_{x\lambda}>0$ (i.e., the signs of $\lambda_{x\parallel}$ and $\lambda_{x\perp}$ are the same), the anisotropy is *irrelevant* at the fixed points of each set. Then the behavior of the RG flows is similar to that for the isotropic case studied by Chattopadhyay, so that we will not discuss this case. In case of $r_{x\lambda}<0$ (i.e., the signs of $\lambda_{x\parallel}$ and $\lambda_{x\perp}$ are opposite), the anisotropy is *relevant* at the fixed points of each set if $d<d_c$. Then the behavior of the RG flows is different from the one for the isotropic case. Although there exists a nontrivial stable fixed point for $r_{x\lambda}<0$, it is difficult to derive the analytic formula for the fixed point r_ν^* and the on- or off-axis fixed points for each set of parameters for $d>2$. Instead, the stable fixed point can be found numerically by iterating the RG transformation. However, we can obtain an analytic formula for r_ν^*, and the on- or off-axis fixed points exactly for $d=2$. From now we consider only the case $r_{x\lambda}<0$ in $d=2$, where the anisotropy is relevant.

In the $(U_{00},U_{\rho 0})$ plane, two fixed points are on axes since $d(U_{00}/U_{\rho 0})/d\ell=-\rho(U_{00}/U_{\rho 0})$. For $\rho=0$, the physical properties of the ANKPZ equation are the same as those of the AKPZ equation with white noise, which was studied by Wolf [15]. The two fixed points $r_\nu^*=\pm r_{0\lambda}$ are obtained exactly for $d=d_c=2$ from Eq. (6), by setting $dr_\nu/d\ell=0$. At the fixed point $r_\nu^*=-r_{0\lambda}$, the parameter U_{00} is renormalized to zero. Hence the nonlinear terms in the AKPZ equation with white noise become irrelevant, and the AKPZ equation belongs to the EW universality class. This means that when the signs of λ's are opposite, the nonlinear terms of the AKPZ equation become irrelevant even though the values of λ's are finite. However, when $\rho>0$, there exists a stable fixed point on the $U_{\rho 0}$ axis if $d<d_c=2+2\rho$. Thus, even though the signs of nonlinearities are opposite, the nonlinear terms do not become irrelevant. When $d=2$, Eq. (6) can be written exactly as

$$\frac{dr_\nu}{d\ell}=\frac{\pi 2^{-\rho}(r_{\rho\lambda}+r_\nu)U_{\rho 0}^2}{8(1+\sqrt{r_\nu})^2 r_\nu^{3/2}}[(1+\sqrt{r_\nu})^2(r_{\rho\lambda}-r_\nu)+6\rho(r_{\rho\lambda}+2r_{\rho\lambda}\sqrt{r_\nu}-2r_\nu^{3/2}-r_\nu^2)]. \tag{16}$$

Thus $r_\nu^*=-r_{\rho\lambda}$ is the solution of $\partial r_\nu/\partial\ell=0$. For $r_{\rho\lambda}<0$, the fixed point is given exactly by

$$U_{00}^{*2}=0,\ \ U_{\rho 0}^{*2}=\frac{2^{3+2\rho}\sqrt{r_\nu^*}\rho}{\pi[3\times 2^\rho(1+6\rho\sqrt{r_\nu^*}/(1+\sqrt{r_\nu^*})^2)-1]}. \tag{17}$$

From Eq. (5), for small ρ we obtained

$$z=2-\rho+O(\rho^2). \tag{18}$$

Thus, we have a stable fixed point at which a new exponent differs from that of isotropic NKPZ equation. When $\rho<0$, since $d=2>2+2\rho$, the ANKPZ equation belongs to the EW universality class. This is the same situation as for the ANKPZ equation with white noise, which was studied by Mukherji and Battacharjee [11].

In the $(U_{0\sigma},U_{\rho\sigma})$ plane, the fixed points are on axes since $d(U_{0\sigma}/U_{\rho\sigma})/d\ell=-\rho(U_{0\sigma}/U_{\rho\sigma})$. When $\sigma>0$ and $\rho=0$, one can see from Eq. (9) that there exists a stable fixed point on the $U_{0\sigma}$ axis if $d<d_c=2+2\sigma$. At this fixed point, where short-range interaction couples with correlated noise, the dynamic and the roughness exponents are obtained from Eq. (5), Eq. (9) and the relation $z+\alpha=2$ by setting $\partial r_\nu/\partial\ell=0$ and $\partial U_{0\sigma}/\partial\ell=0$,

$$z=\frac{1}{3}(4+d-2\sigma),\quad \alpha=\frac{1}{3}(2-d+2\sigma), \tag{19}$$

provided that $\nu_\perp\neq 0$ and $U_{0\sigma}\neq 0$. Equation (19) shows that the roughness exponent increases while the dynamic exponent decreases, due to the presence of spatial correlations in noise. When $d=2$, we obtain $r_\nu^*=-r_{0\lambda}$ as the solution of $\partial r_\nu/\partial\ell=0$. For $r_{0\lambda}<0$, the stable fixed points of the parameters are given exactly by

$$U_{0\sigma}^{*2}=\frac{2^3\sqrt{r_\nu^*}\sigma}{\pi(3+\zeta)},\quad U_{\rho\sigma}^{*2}=0, \tag{20}$$

where

$$\zeta=\frac{12\sigma\sqrt{r_\nu^*}}{(1+\sqrt{r_\nu^*})^2}. \tag{21}$$

Thus, if $d<d_c=2+2\sigma$, $\sigma>0$, and $\rho=0$, this fixed point is stable, and the exponents are given by Eq. (19) with $d=2$.

When $\rho>0$, a stable fixed point on the $U_{\rho\sigma}$ axis becomes relevant if $d<d_c=2+2\rho+2\sigma$. From Eq. (5), Eq. (10), and $z+\alpha=2-\rho$, the dynamic and roughness exponents are given by

$$z=\frac{1}{3}(4+d-2\rho-2\sigma),\quad \alpha=\frac{1}{3}(2-d+2\sigma-\rho), \tag{22}$$

provided that $\nu_\perp\neq 0$ and $U_{\rho\sigma}\neq 0$. In this case, the correlated noise coupled with the long range KPZ nonlinearity decreases the dynamic exponent. However, unlike Eq. (19), the roughness exponent depends on *both values* of ρ and σ. For $2\sigma+2-d<\rho$ the roughness exponent decreases, while for $2\sigma+2-d>\rho$ it increases. But if $2\sigma=\rho$, the roughness exponent does not depend on the correlated noise as well as the long-range KPZ nonlinearity. A solution of $\partial r_\nu/\partial\ell=0$ is given by $r_\nu^*=-r_{\rho\lambda}$ in $d=2$. For $r_{\rho\lambda}<0$, the stable fixed points are given exactly by

$$U_{0\sigma}^{*2}=0,\quad U_{\rho\sigma}^{*2}=\frac{2^{3+\rho}\sqrt{r_\nu^*}(\rho+\sigma)}{\pi[3+(1+3\rho/2\sigma)\zeta]}. \tag{23}$$

Thus, if $\rho+\sigma>0$ and $\rho>0$, this fixed point is stable, and the exponents are given by Eq. (22) with $d=2$. In contrast, for $\rho<0$, the fixed point Eq. (23) becomes irrelevant except for $\lambda_0=0$. The surface in all space $(U_{0\sigma},U_{\rho\sigma})$ is governed by the fixed point [Eq. (20)].

In the $(U_{00},U_{0\sigma})$ plane, the physical properties of the ANKPZ equation are the same as those of the AKPZ equation with spatially correlated noise [18]. In this plane the dynamics of the AKPZ equation with correlated noise is different from that of the AKPZ equation with white noise, which belongs to the EW universality class. When $r_{0\lambda}<0$, a stable fixed point of the RG flows exists in the off-axis region, provided that $d<d_c=2+2\sigma$ and $\sigma>0$. At $r_\nu^*=-r_{0\lambda}$ with negative value of $r_{0\lambda}$, the fixed point of the parameters is given exactly in $d=2$ by

$$U_{00}^{*2}=\frac{4\sigma\sqrt{r_\nu^*}}{\pi}\left(\frac{\zeta^2+4\zeta+9-(\zeta+3)\sqrt{\zeta^2+2\zeta+9}}{\zeta^2}\right),$$

$$U_{0\sigma}^{*2}=\frac{4\sigma\sqrt{r_\nu^*}}{\pi}\left(\frac{-\zeta-9+3\sqrt{\zeta^2+2\zeta+9}}{\zeta^2}\right). \quad (24)$$

The dynamic exponent and the roughness exponents associated with the fixed point are given by Eq. (19), which is valid for $d<d_c=2+2\sigma$ and $\sigma>0$. Although these exponents are the same as those obtained from the isotropic KPZ equation with spatially correlated noise, there is a difference in the range of applicability of Eq. (19). For the isotropic case, Eq. (19) is valid only for $d<3/2$ and $\sigma_{min}<\sigma<\sigma_{max}$, where $\sigma_{min}=d(d-2)/[8(d-3/2)]$ and $\sigma_{max}=(d+1)/2$ [19]. However, for the anisotropic case, Eq. (19) is valid for $d<d_c=2+2\sigma$ and $\sigma>0$. When $d>d_c=2+2\sigma$, the effect of a nonlinear term becomes irrelevant, and the dynamics of the ANKPZ equation is the same as that of the EW equation.

In the $(U_{\rho 0},U_{\rho\sigma})$ plane, the stable fixed point exists in the off-axis region of the parameter space for $d<d_c\equiv 2+2\rho+2\sigma$. When $d=2$ and $\sigma>0$, solving $\partial U_{\rho 0}/\partial \ell=0$ and $\partial U_{\rho\sigma}/\partial\ell=0$ with $r_\nu^*=-r_{\rho\lambda}$, which is solution of $\partial r_\nu/\partial\ell=0$, we obtain a stable fixed point for $r_{\rho\lambda}<0$. The stable fixed point is located at

$$\begin{aligned}U_{\rho 0}^{*2}=\frac{\sqrt{r_\nu^*}}{\pi\sigma\zeta^2}\{&36\sigma^2+\zeta^2(3\rho+2\sigma)^2-4\zeta\sigma(-9\rho+2^{1+\rho}\rho\\&-6\sigma+2^{1+\rho}\sigma)-(3\zeta\rho+6\sigma+2\zeta\sigma)\\&\times(36\sigma^2+\zeta^2(3\rho+2\sigma)^2\\&-4\zeta\sigma[(-9+2^{2+\rho})\rho+2(-3+2^{1+\rho})\sigma])^{1/2}\},\end{aligned} \quad (25)$$

$$\begin{aligned}U_{\rho\sigma}^{*2}=\frac{\sqrt{r_\nu^*}}{\pi\sigma\zeta^2}\{&-36\sigma^2-3\zeta^2\rho(3\rho+2\sigma)\\&+4\zeta\sigma(-9\rho+2^{1+\rho}\rho-3\sigma+2^{1+\rho}\sigma)\\&+3(\zeta\rho+2\sigma)(36\sigma^2+\zeta^2(3\rho+2\sigma)^2\\&-4\zeta\sigma[(-9+2^{2+\rho})\rho+2(-3+2^{1+\rho})\sigma])^{1/2}\}.\end{aligned}$$

This fixed point is stable for $\rho_{min}<\rho<\rho_{max}$ and $d<2+2\rho+2\sigma$, where $\rho_{min}=-\sigma$, and ρ_{max} is the value satisfying the condition

$$\frac{2^\rho\rho}{(2^\rho-3)-\dfrac{18\sqrt{r_\nu^*}\rho}{(1+\sqrt{r_\nu^*})^2}}=-\sigma \quad (\rho>0). \quad (26)$$

At this fixed point, the dynamic and roughness exponents are given by Eq. (22) with $d=2$. As ρ goes to zero, the fixed point of Eq. (25) moves to the fixed point of Eq. (24), and the case of short-ranged interaction is recovered.

In conclusion, we have studied the ANKPZ equation with spatially correlated noise in several parameter spaces, by using the one-loop dynamic RG transformation. When the signs of nonlinearity $\lambda_{0\parallel}$ and $\lambda_{0\perp}$ or $\lambda_{\rho\parallel}$ and $\lambda_{\rho\perp}$ are the same, the anisotropy becomes irrelevant. The scaling property is then described by the isotropic NKPZ equation with correlated noise. However, when the signs of λ's are opposite, unlike the case of short-ranged interaction and white noise, the stable non-KPZ fixed point exists in the on- or off-axis of the four-dimensional parameter spaces of the effective coupling constants. At the stable fixed point, the scaling behavior of the NAKPZ equation with white noise is different from that of the isotropic NKPZ equation with white noise. However, for the correlated noise, the same exponents are given in both NKPZ and NAKPZ equations. This is because both correlated noise and λ terms are not renormalized.

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